Economics of Politics, Problem set #2

Instructor: Hideki Konishi

Exercise 1.

Two parties, L and R, are competing for a single seat in an election. The policy space is one-dimensional, i.e., $p \in [0,1]$, and we will denote party i's campaign promise by p_i. Parties have their own preferences over policies represented by their continuous utility functions, $U_i(p)$ for $i = L, R$. The utility functions are single-peaked, and $U_L(p)$ is maximized at $p = 0$ while $U_R(p)$ is at $p = 1$. Voters also have single-peaked preferences represented by $V_j(p) = -(p - p_j)^2$, where p_j is voter j's most-preferred policy and it distributes over $[0,1]$ following a continuous cumulative distribution function, $F(p)$. That is, $F(p)$ shows the share of voters in the population with $p_j \leq p$. Its median, denoted by p_m, satisfies $0 < p_m < 1$.

In the election, the parties simultaneously commit themselves to their campaign promises and all voters vote sincerely for either party. When a tie occurs, the winner is selected randomly with probability 1/2. Answer the following questions.

(1) Show that $p_L = p_R = p_m$ occurs in a Nash equilibrium.

(2) Does $p_L < p_R \leq p_m$ occur in a Nah equilibrium, and why?

(3) Suppose that p_L and p_R ($p_L < p_R$) are policies between which the median voter is indifferent. Does such a pair of policies constitute a Nash equilibrium, and why?

Exercise 2.

Consider the same electoral competition in exercise 1. Now the distribution of voters' preferences are uncertain from the point of the two parties. They know that the median voter's most-preferred policy, p_m, is a random draw from a continuous cumulative distribution function, $H(p)$, which represents the probability of $p_m \leq p$. The probability density function is denoted by $h(p)$. Answer the following questions.

(1) Suppose that the two parties promised p_L and p_R such that $p_L < p_R$. Then, formulate party L's expected payoff function.

(2) Considering the first order condition for party L's payoff maximization, show that $p_L = p_R$ does not occur in a Nash equilibrium.

(3) Suppose that $H(p)$ is a uniform distribution with support $[0,1]$ and that the parties' utility functions are $U_L = -p^2$ and $U_R = -(p-1)^2$. What platforms will the respective parties adopt in a Nash equilibrium?

Exercise 3.

Consider the following two-period game of electoral competition. At the beginning of period 1, an election is held, where two male candidates, A and B, announce effort levels, e_A and e_B, and voters vote for either candidate. A tie is randomly resolved. Then, the winner, who is indexed by I, takes office and chooses an effort level, e_I, which he can change freely from what he announced in the election. At the beginning of period 2, Nature chooses a challenger, who is female with probability 1/2, and an election is held, in which the incumbent and the challenger run for office. The voters always prefer a female candidate to a male, but are indifferent to male candidates. A tie is also resolved randomly. The winner makes no effort in office in period 2. Making effort of e in period 1 costs a politician by $c(e) = e^2$, while holding office for a period gives ego rents of $R > 0$. The discount factor applied to the payoff in period 2 is $\delta \in (0, 1)$, and the reservation payoff to politicians is equal to zero.

Suppose that voters can communicate to coordinate their voting decisions in period 2 and adopt a retrospective voting strategy, in which they will reelect the incumbent for sure if $e_I \geq \overline{e}$ and otherwise vote for the challenger.

(1) If voters can commit to such a voting strategy before the winner in the first period election takes office, then what is the optimal level of $\overline{e}$ for them?

(2) The above voting strategy is not credible for the incumbent politician in period 1. Why?

(3) Specify a retrospective voting strategy that is credible to the incumbent in period 1, and solve for the maximum amount of effort that voters can induce him to take.

Suppose that voters cannot communicate to coordinate their decisions on $\overline{e}$ in the retrospective voting strategy, but instead they replace $\overline{e}$ with what the incumbent announced in the election he won.

(4) Consider the election in period 1. Given e_B, what is the probability of candidate A winning the election when announcing e_A?

(5) Show that the maximum amount of effort obtained in (3) with voter communication is realized in the subgame-perfect Nash equilibrium.

Exercise 4.

Consider a common agency game consisting of three SIGs, in which the government chooses a policy $p \in \{Y, N\}$ and SIG $i = 1, 2, 3$ enjoy benefits π_{pi} when the government chooses p. The contribution schedule offered by SGI i is $r_i = (c_{Yi}, c_{Ni})$, where $c_{pi} \geq 0$ is the amount of money it will give to the government when p is chosen. Given p and r_i, SIG i's payoff is $\pi_{pi} - c_{pi}$. The government cares only for the sum of the money that the SIGs will give. Prove that only the policy that maximizes $\pi_{p1} + \pi_{p2} + \pi_{p3}$ is implemented in the subgame-perfect Nash equilibrium.