Economics of Politics, Problem set #1

Instructor: Hideki Konishi

Exercise 1.

Three voters, 1, 2, and 3, are voting over a consumption tax rate, t_c. The set of alternatives is $\mathcal{A} = \{0\%, 5\%, 10\%\}$. $V_i(t)$ denotes voter i's utility when the tax rate is $t_c\%$. Specifically,

$$V_1(0) > V_1(5) > V_1(10)$$
$$V_2(5) > V_2(10) > V_2(0)$$
$$V_3(10) > V_3(0) > V_3(5)$$

Answer the following questions.

(1) Does a Condorcet winner exist? If it does, which alternative is a Condorcet winner?

(2) Does each voter have single-peaked preferences? If not, whose preferences are not single-peaked?

Exercise 2.

Three voters, 1, 2, and 3, are voting over a pair of social security expenditures, s, and military expenditures, m. Voter i's utility, $V_i(s, m)$, is specified as:

$$V_i(s, m) = -(s - s_i)^2 - (m - m_i)^2,$$

where (s_i, m_i) is voter i's most-preferred pair of s and m. The set of alternatives is given as $\mathcal{A} = \{(s, m) \in \Re^2 \mid s + m \leq 100,\ s \geq 0,\ m \geq 0\}$. Note that $(s, m) \in \Re^2$ means that both s and m are real numbers. Answer the following questions.

(1) Suppose that $(s_1, m_1) = (0, 0)$, $(s_2, m_2) = (100, 0)$, and $(s_3, m_3) = (0, 100)$. Does there exist a Condorcet winner? If it does, what pair of s and m is the one? If it does not, explain why.

(2) Suppose that $(s_2, m_2) = (100, 0)$, and $(s_3, m_3) = (0, 100)$ as in the previous question. What condition should (s_1, m_1) satisfy to guarantee the existence of a Condorcet winner?

Exercise. 3

Consider a game in which two individuals, $i = 1, 2$, simultaneously provide a pure public good. Individual i's utility function is

$$u_i = 100x_i - 1.25x_i^2 + c_i,$$

where x_i is i's consumption of the public good and c_i is her consumption of a private good. Let y_i denote individual i's provision of the public good, which must satisfy $y_i \geq 0$, and w_i be her initial endowment of the private good. The unit cost of a public good provision is equal to 50. Answer the following questions, assuming that w_i is sufficiently large.

(1) How are x_1, x_2, y_1 and y_2 related? Show equations holding among them.

(2) Solve the Pareto-efficient provision of the public good, assuming an internal solution.

(3) Given individual j's provision of the public good, y_j, What is individual i's best response to it?

(4) Solve all the (pure-strategy) Nash equilibrium of this game, and show that the public good is socially undersupplied in the equilibria.

(5) Explain the reason for the equilibrium underprovision of the public good, using the two words, "social marginal benefits" and "private marginal benefits."

(6) Suppose instead that individual 2 chooses y_2 after individual 1 chooses y_1, knowing the amount of y_1. How much does individual 1 provide in the subgame-perfect Nash equilibrium of this game?

Exercise 4. Consider a society consisting of n individuals indexed by $i = 1, 2, \cdots, n$ $(n \geq 3)$. Individual i has an endowment of a private good, w_i, one unit of which can be transformed into one unit of a pure public good. Each individual has a common utility function,

$$u_i = \alpha \log c_i + (1 - \alpha) \log g, \ 0 < \alpha < 1,$$

where c_i is her consumption of the private good and g is that of the public good. Answer the following questions.

(1) Let g^o be the Pareto-efficient provision of the pure public goods. Formulate the maximization problem to solve g^o.

(2) Show that $g^o = (1-\alpha)\sum_{i=1}^{n} w_i$, manipulating the first order conditions in the maximization problem formulated in (3).

Suppose that individuals voluntarily provide a public good out of their endowment. Let g_i be individual i's voluntary contribution and g_{-i} be the sum of the others' contributions, i.e., $g_{-i} = \sum_{j \neq i} g_i$.

(3) What is individual i's optimal contribution when g_{-i} is taken as given?

(4) Consider a Nash equilibrium in which every individual makes a positive contribution, i.e., $g_i > 0$ for all i. What is the equilibrium total provision of the public good?

(5) Under what condition does a Nash equilibrium exist with every individual contributing a positive amount?

Suppose next that a majority voting takes place to determine the provision of the public good. Once the provision is determined, each individual has to share the cost equally; that is, if g units of the public good are supplied, each has to pay g/n units of the private good.

(6) Because of the cost-sharing rule, individual i's consumption of the private good is equal to $w_i - g/n$ when g units of the public goods is provided (as far as $w_i \geq g/n$). What is individual i's most-preferred amount of the public good?

(7) Show that every individual has single-peaked preferences over the provisions of the public good by making use of the second order derivative of her utility function.

(8) Suppose that n is odd and $w_1 < w_2 < \cdots < w_n$. What is the Condorcet winner?

(9) Under what condition does the Condorcet winner coincides with the Pareto-efficient level, g^o?